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FLOW INTO A STRATIFIED RESERVOIR

William M. Thurston

California University

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16. Abstract: The flow caused by a line source discharging into a stagnant, linearly density stratified reservoir was studied in a tank. The flow enters the reservoir as a horizontal line jet but immediately passes through an internal hydraulic jump and forms a slowly moving wedge of fluid mixed into the injection by the jump. Ahead of this mixed layer the inflow pushes a wide layer termed the entering layer, which extends to the opposite end of the reservoir and consists of fluid already in the reservoir before the jet. The inflow also induces a series of layers of flow in alternating directions above and below the entering layer. The mixed layer was made visible by mixing blue dye into the supply fluid. The length, thickness, and tip speed of the mixed layer were measured as a function of time, and an empirical scaling relationship was derived to relate the differing experimental conditions. Use of the scaling factors allows the results to be applied to prototype reservoirs to predict the extent of mixed layers which might occur during the pumping phase in a jump-storage reservoir. (Knapp-USGS)

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FLOW INTO A STRATIFIED RESERVOIR

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ABSTRACT

This report describes the results of an experimental study of the flow caused by a line source discharging into a stagnant, linearly density stratified reservoir. The flow enters the reservoir as a horizontal line jet but immediately passes through an internal hydraulic jump and forms a slowly moving wedge of fluid composed partly of the injected fluid and partly of fluid mixed into the injection by the jump. Ahead of this mixed layer the inflow pushes a wide layer termed the entering layer, which extends to the opposite end of the reservoir and consists of fluid already in the reservoir before the jet was begun. The inflow also induces a series of layers of flow in alternating directions above and below the entering layer.

Experiments are described in which the mixed layer was made visible by mixing blue dye into the supply fluid. The length, thickness, and tip speed of the mixed layer were measured as a function of time, and an empirical scaling relationship was derived to relate the differing experimental conditions. Use of the scaling factors allows the results to be applied to prototype reservoirs to predict the extent of mixed layers which might occur, for instance, during the pumping phase in a pump-storage reservoir.

This report was submitted in fulfillment of Project Number 15040 EJZ, under the sponsorship of the Water Quality Office, Environmental Protection Agency.

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SECTION I

CONCLUSIONS

An experimental study was made of the effects of an entering discharge into a stagnant, density-stratified reservoir. The discharge entered as a line jet with the same density as the surrounding fluid at the discharge elevation. The jet immediately passed through an internal hydraulic jump, after which the mixed flow of entering and entrained fluid moved forward as a wedge. With respect to this mixed layer the following conclusions have been established:

1. The maximum scaled horizontal velocity of the front tip of the mixed layer in a tank of infinite length may be given by

$$U^*_{max} = 3/4K\ (t^*)^{-1/4}$$

where $K = 3.16 \times 10^{-2}$ and U^* and t^* are scaled velocity and time.

2. The thickness of the mixed layer may be given by:

$$Y/Y_m = 1 - (x/x_f)^{1/2}$$

3. The scaled entrainment may be given by:

$$E^* = K_1\ (t^*)^{3/4}$$

where K_1 depends on Froude number at inlet conditions.

4. The appropriate scaling factors for time and horizontal distance are

$$t^* = t(\varepsilon g)^{1/2}$$

$$x^* = x(\varepsilon g)^{5/12}\ \nu^{11/18}/q\ g^{2/9}$$

SECTION II

RECOMMENDATIONS

The following extensions of this study are recommended:

a) This study was conducted using the simplest geometry. Future studies might include: 1) sloping bottom, 2) different locations of source, 3) more complex, characteristic boundaries.

b) A study of the general current pattern induced by the source throughout the entire model should be made which should include the velocity profiles, streamlines, etc. to provide information on the rate of vertical mixing throughout the reservoir.

c) Both steady and unsteady current patterns should be investigated for the model, including the retarding effects of the end wall.

d) Future studies should include buoyant sources and three-dimensional sources.

e) Theoretical and experimental investigations of the internal jump formed near the source and correlation with the inlet densimetric Froude number should be made.

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SECTION III

INTRODUCTION

The traditional role of surface water reservoirs is as a storage basin for a given quantity of water, primarily for water supply or flood control. Only in recent years have engineers become aware that reservoirs also play an important and delicate role in the management of water quality. Properly managed, reservoirs can provide a useful means of regulating the quality of water in a stream, including its temperature, dissolved oxygen, and other constituents, in whatever way will be most beneficial to the downstream users. Improperly managed a reservoir can contribute to the destruction of a river environment by permanently changing its temperature, oxygen content, and turbidity. Much remains to be learned about how the flow in a reservoir can be controlled to produce the desired quality of outflow. In the report describing the research conducted by this project for the previous year, Imberger and Fischer (1970) described the withdrawal layer which forms when water is removed from a stratified reservoir. The present report describes the reverse problem, the flow which results when water is injected into a stratified reservoir.

The injection problem is important in two ways. Firstly, the research described herein is directly applicable to water quality management in reservoirs used as part of pump-storage projects, in which water is pumped into the reservoir during a part of the cycle. Secondly, the flow patterns observed in this experiment are likely to be similar to the flow caused by a river discharge into a reservoir, because the river discharge normally sinks to a level equal to its own density before flowing in a layer into the reservoir. The total flow created by the inflow from a river and the discharge through the dam is more complex than the one studied here, but the study presented herein may be seen as a first step in identifying the flow patterns caused by an injection into a stratified reservoir. A complete understanding of the mechanics of stratified flow in reservoirs and the resulting effects on water quality management remains a long term goal which will require considerable further study.

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SECTION IV

DESCRIPTION OF THE EXPERIMENT

The Experimental Tank

The experiments were performed in a 42 feet long by 4.00 feet deep by 1.25 feet wide tank located at the University of California Richmond Field Station. The walls and bottom were constructed of tempered plate glass set in a steel framework. The tank is shown schematically in Figure 1 and photographically in Figure 2.

Two trolleys were placed on rails on top of the tank. One served as a mounting platform for the stratifying apparatus while the other held the salinity probe and the associated vacuum pump. Mounting the salinity probe on the trolley allowed the same probe to be used to measure both the density variations of the receiving fluid and the density of the supply fluid. The injection system was located at one end of the tank. Vertical measuring tapes were installed at 22 predetermined points measured horizontally from the injection end of the tank.

The Salinity Probe

Thompson (1968) gives a detailed explanation of design of the type of conductivity probe used throughout this study.

The probe shown in Figure 3 was calibrated at the beginning of every run. The calibration was sensitive to the water temperature. Standard solutions whose specific gravities were measured with a hydrometer, were prepared and used to calibrate the probe after these solutions had reached the temperature of the water in the tank; this was accomplished by stirring the solutions since most of the time their temperature was lower than that of the water in the tank.

The difference between the temperature of tank water and the temperature at which the specific gravities were measured was always less than 2°C. Since only the density gradient of the tank water is required, no compensation was made for this temperature difference. Furthermore, the temperature distribution of the water inside the tank was uniform with depth.

Figure 4 shows a typical calibration curve. The calibration curves were nearly linear, with a slight decrease in sensitivity for the larger concentrations. Data were taken with a Brush Dual strain gage

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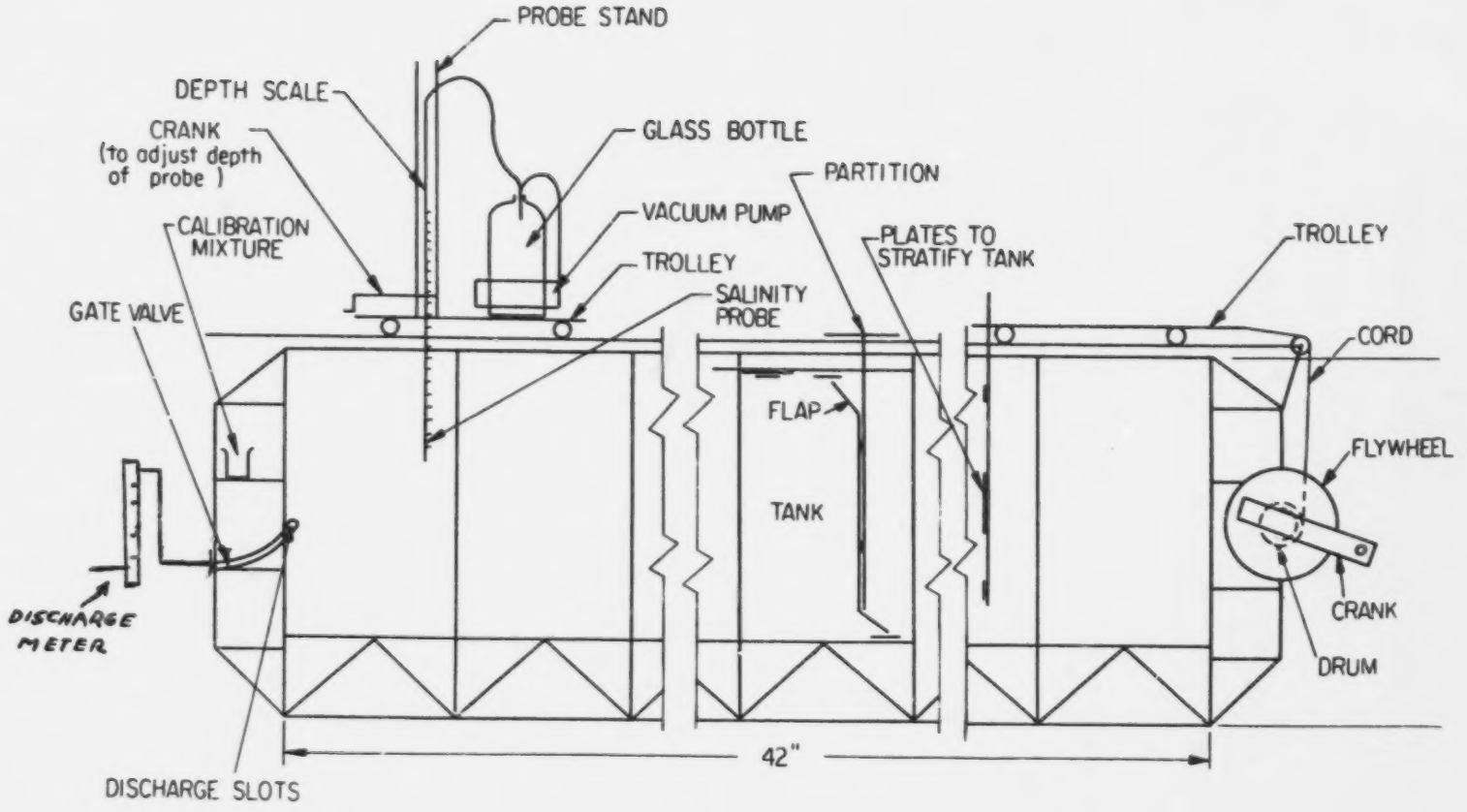

FIG. 1 GENERAL LAYOUT OF EXPERIMENTAL TANK

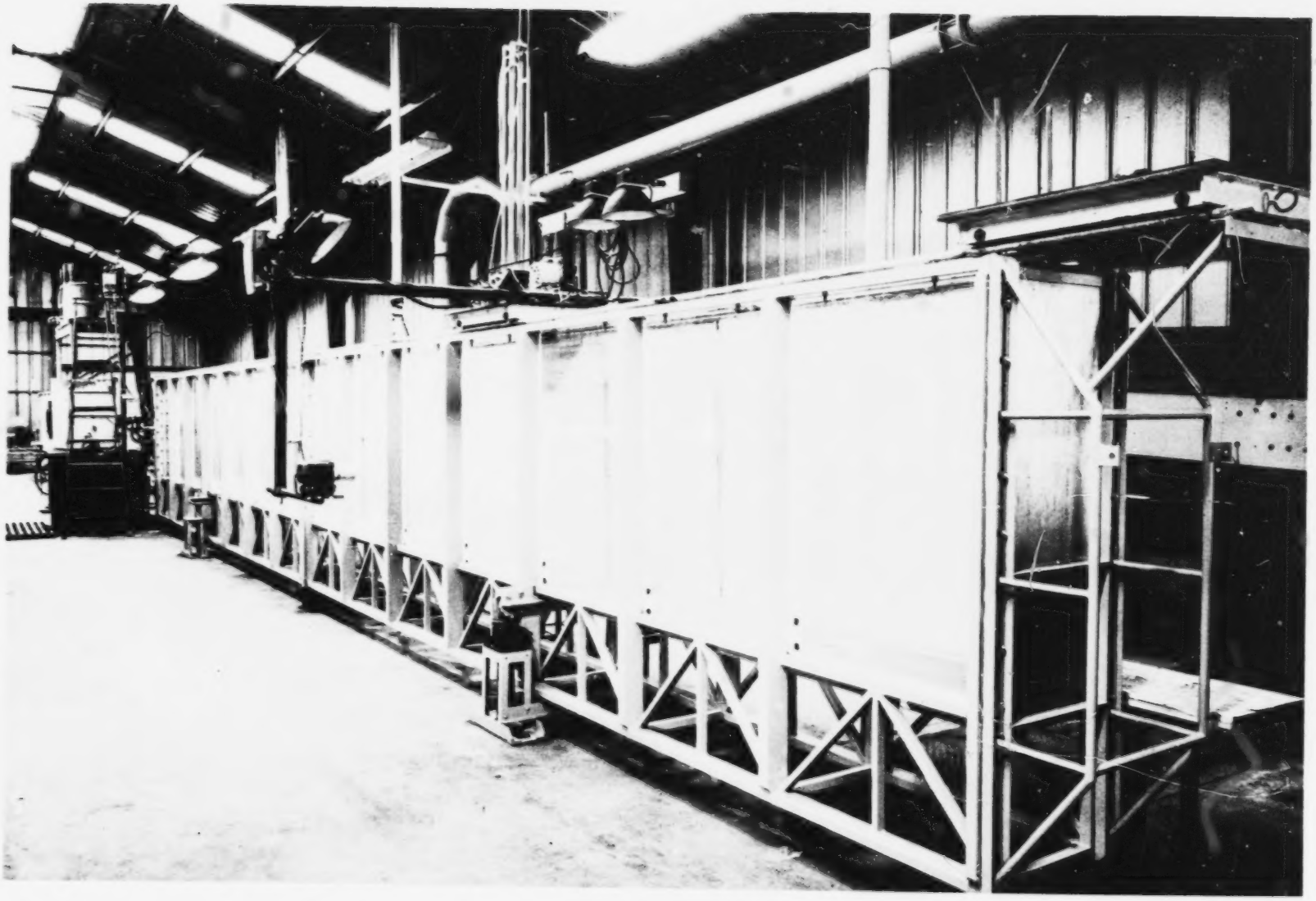

FIG. 2 EXPERIMENTAL TANK

FIG. 3 CONDUCTIVITY PROBE

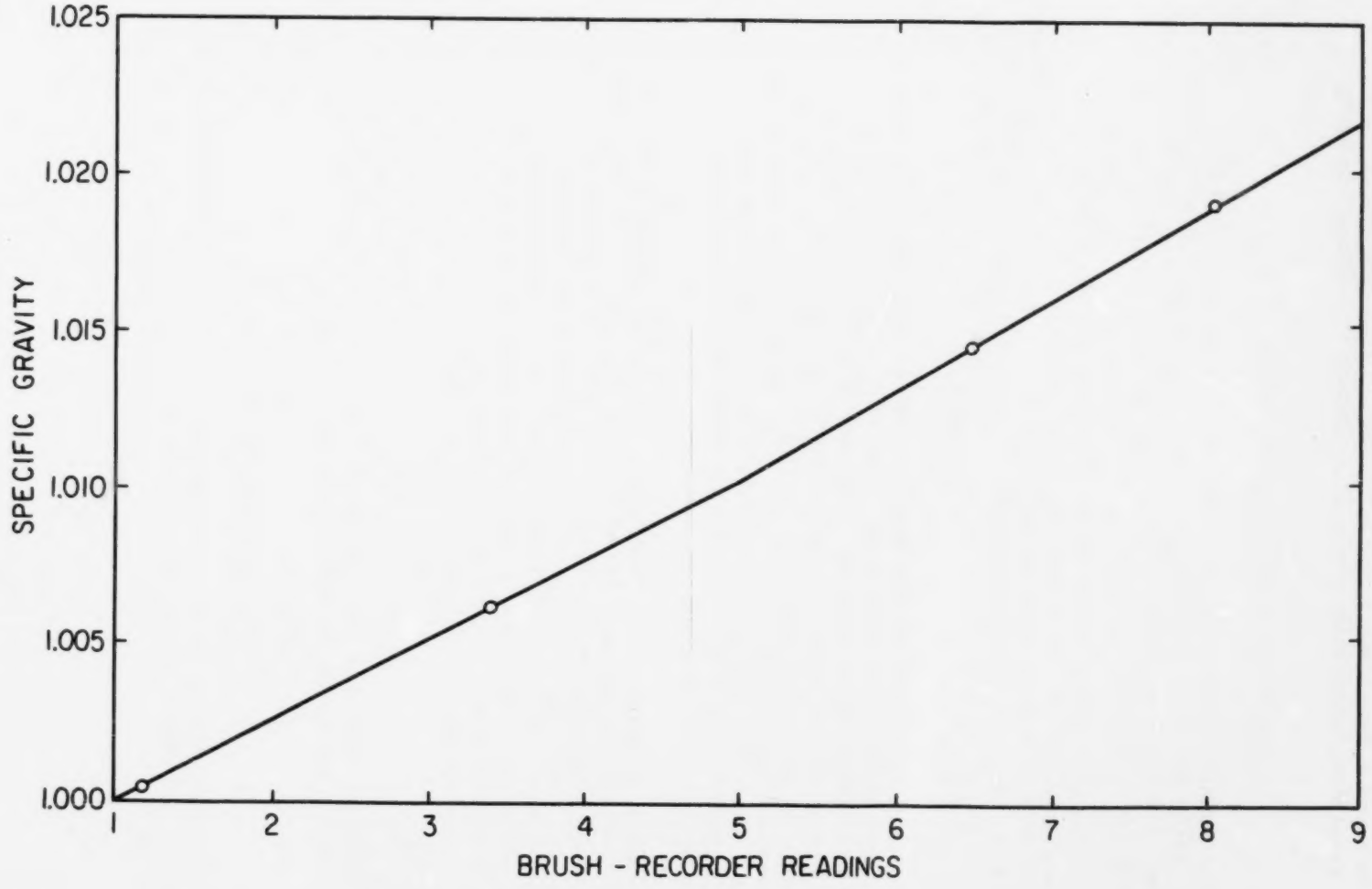

FIG. 4 TYPICAL CALIBRATION CURVE

amplifier and a Brush Mark 220 recorder. Drift, presumably due to the amplifier, frequently made it necessary to balance the bridge.

Establishment of a Linear Density Profile

A linear density profile can be established by several methods. Slotta (1969) has used a special technique to obtain a linear density profile. However the authors have chosen the technique reported by Clark et al. (1967) and successfully used by Imberger (1970).

The tank was first filled with fresh water, which was allowed to come to room temperature. A gated partition consisting of a piece of plywood with 9" hinged flaps at the bottom and top, was then introduced vertically into the center of the tank. The partition was sealed to the glass walls with a soft rubber spacer. This was sufficient to stop any transfer of water from one side to the other.

An appropriate amount of salt was then placed and thoroughly mixed in one half of the partitioned tank. The salt used was an ordinary kiln dried fine commercial product. The uniform mixture was allowed to come to rest, while some of the entrapped air surfaced. The flaps of the partition were then opened very slightly to allow the salt water to flow beneath the fresh water and the fresh water to flow over the salt water. About one half hour elapsed before the whole tank had a two layer stratification in it with a sharp interface between them. The partition was then carefully removed.

To obtain a linear profile, a series of two plates, mounted on the trolley, were then briskly moved through the interface with a crank and pulley arrangement at the end of the tank. The largest plate moved directly through the interface while the smaller one stirred up the bottom layer. The wakes behind the plates created sufficient turbulence to allow mixing. After internal oscillations ceased, a density profile measurement was taken with the conductivity probe. If the profile was not quite linear then a broom was moved up and down all along the tank. A typical density profile is given in Figure 5. The stratification procedure took one day, including washing the tank, filling it and preparing the linear profile. Once the desirable profile was obtained, the tank was allowed to stand overnight, ensuring a static condition for the run the following morning.

The Injection System

The injection system is shown schematically in Figure 6 and the main elements are described in the following lines.

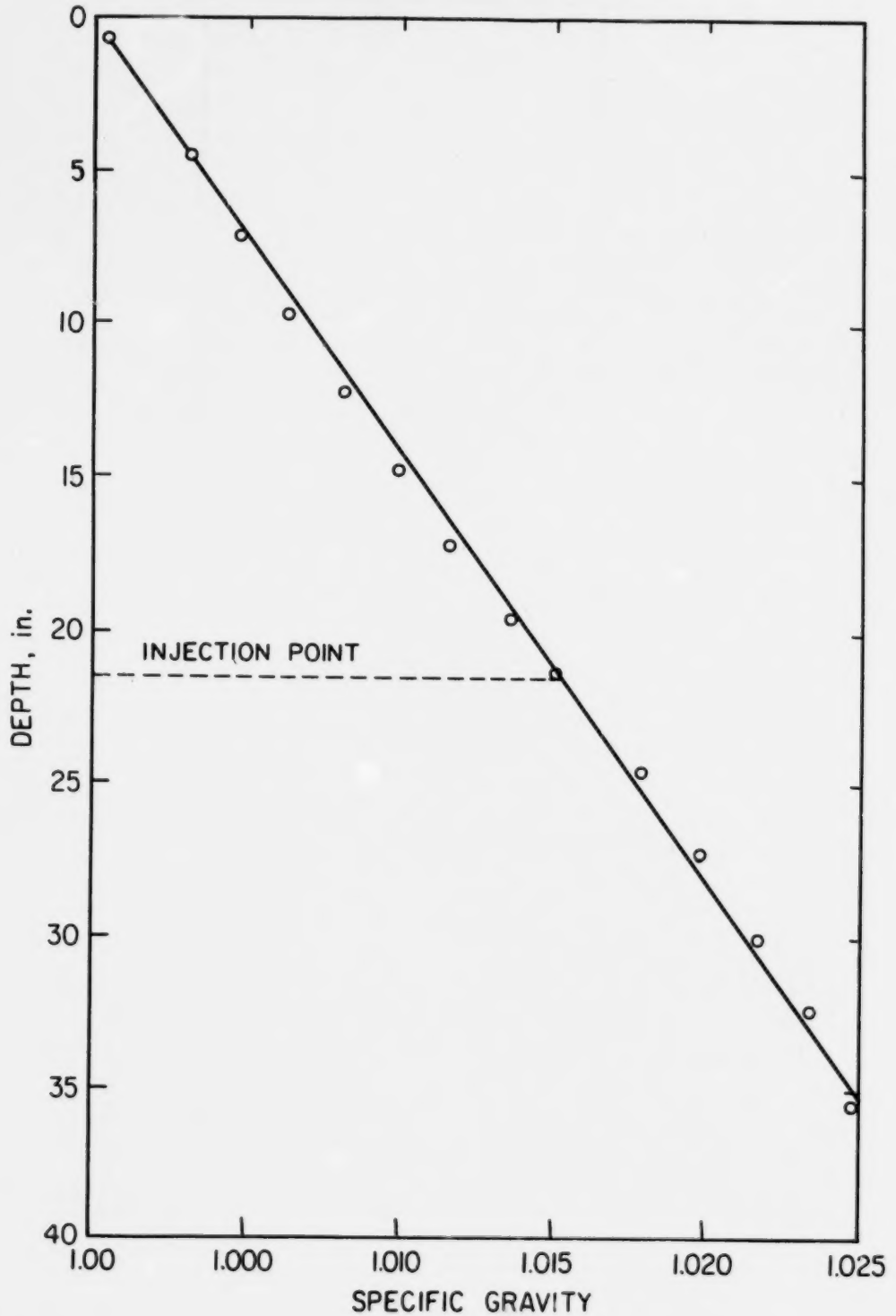

FIG. 5 TYPICAL DENSITY PROFILE

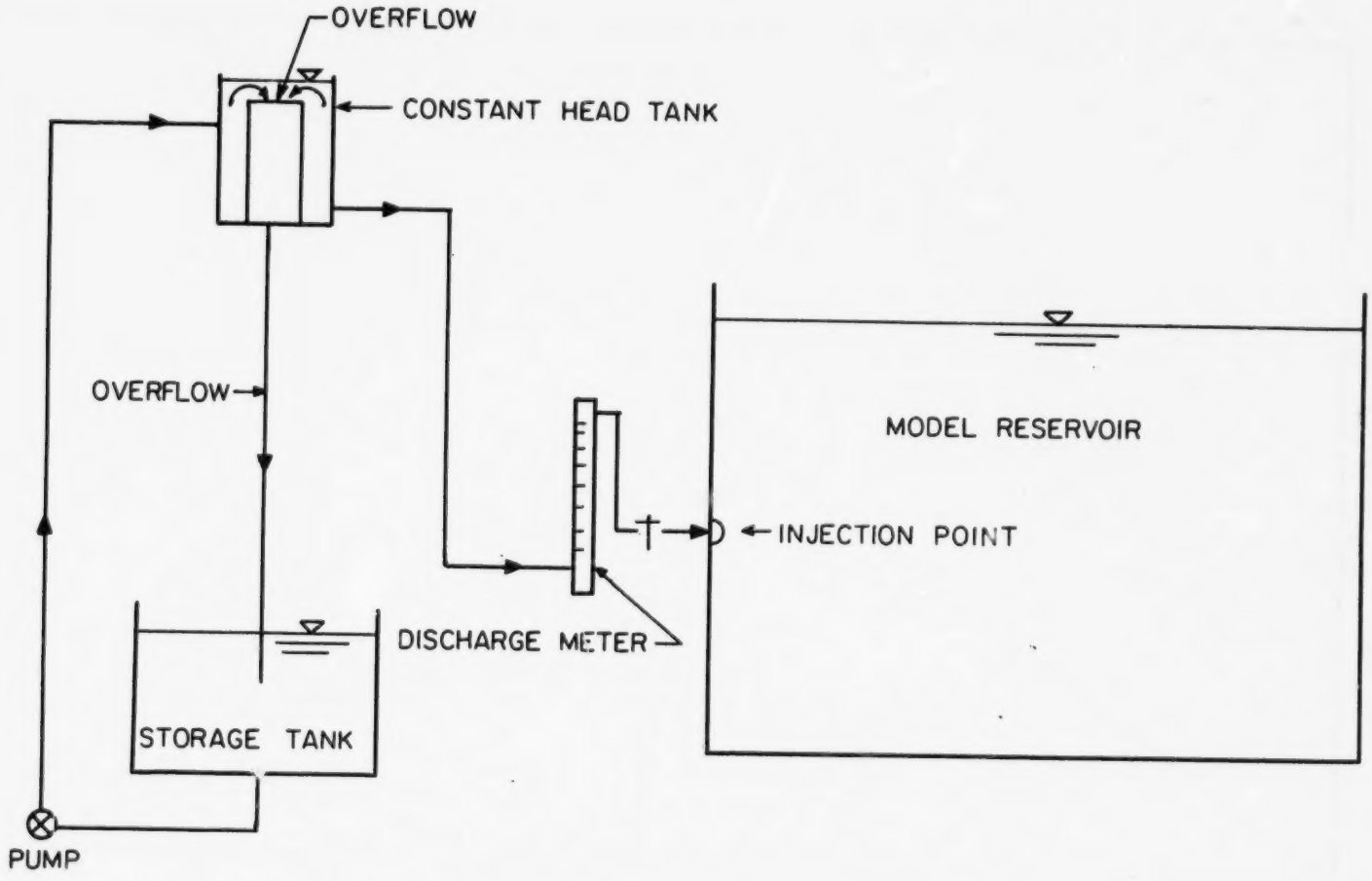

FIG. 6 INJECTION SYSTEM

The storage tank is a cylindrical tank made out of stainless steel. Dimensions are 4 feet diameter by 2. 5 feet deep. The bottom of the storage tank has a 0. 75 inch opening. The bottom opening is connected to the suction pump through a 0. 75 inch plastic hose. At the top of the tank is the terminal end of a hose that brings into the storage tank the overflow from the constant head tank, so that the solution that is not injected into the model tank is recycled.

The suction pump is of the centrifuge type and it is moved by a 1. 5 hp AC motor. The water is pumped into the constant head tank which is 10 feet above the storage tank.

The constant head tank is required so that the entering discharge into the model tank can be kept constant for the full length of each run. A 0. 75 inch plastic hose carries the chosen discharge from the constant head tank into the discharge meter. This hose has two filters: one near the constant head tank and the other near the entrance to the discharge meter. Both filters were needed to keep larger particles from going into the discharge meter. The filters could be removed from the main line so they could be cleaned. The filter screens had openings of 2 square mm and were cleaned before each run.

Measurements of flow rates were made by means of a rotameter. The rotameter, originally calibrated for sea water with a specific gravity of 1. 025, had to be re-calibrated for the range of specific gravity values used in this study. Although the re-calibration indicates a small density influence on the flow rate, it was small enough relative to the error inherent in reading the rotameter that it may be ignored. The rotameter had a maximum capacity of 95 gph providing a good range for the chosen discharge values.

The entering device consists of a 3/4 inch diameter stainless steel pipe, 35 inches long overall with a continuous horizontal 1/32 inch deep slot which extends over the width of model reservoir. The entering device is located half way between the top and the bottom of the model tank, and discharges horizontally into the tank. In all the experiments the inlet depth was 50 cm below the water surface.

Experimental Procedure

After the linear profile was obtained, the specific gravity at the injection level was measured in the model tank. A solution of salty water with a specific gravity equal to that at the injection level in the storage tank was prepared. This solution was colored with A-Concentrate Blue Dye purchased from Chroma-color Company, Hollywood, California.

The temperature of the solution in the storage tank was kept within 2°C of the temperature of the water in the model tank. A continuous recycle was then allowed for about 15 minutes to assure a good mixed colored solution. Then the solution was allowed to enter into the density stratified fluid by opening an adjusting valve until the desired discharge was obtained.

As the front tip of mixed layer reached each one of the stations a mark was made on the running tape of the Brush Mark 220 recorder. The tape was running at a velocity of 1 mm per sec. The time taken by the front tip to reach each station was measured by taking the length between marks.

The mixed layer profile was measured at various times by reading the measuring tapes. The general current pattern in the model reservoir was observed by dropping into the model reservoir little crystals of potassium permanganate.

Details of Experimental Procedure

1. The model tank was filled with tap water and allowed to stand for several hours.

2. The wooden partition was inserted at the center of the tank.

3. The required amount of salt was inserted into one side.

4. The salt was mixed by agitating the water with a push-broom until uniform. After the mixture was uniform it was allowed to come to rest.

5. The flaps on the partition were opened about 1 in. After half an hour the partition was removed, and the interface was allowed to come to rest.

6. The stratification plates were pulled through the interface.

7. Calibration solutions were prepared and their temperature was checked against that of the model tank water.

8. The probe was calibrated and then used to obtain a density profile of the model tank water.

9. If the density profile was not approximately linear then a broom was carefully moved up and down at several points along the model tank. The stratified fluid was allowed to stand overnight.

10. The specific gravity of the salt water in the model tank was measured at the level of injection. A solution of the same specific gravity was then prepared in the storage tank.

11. The storage tank solution was colored with deep blue dye. Ten spoons were usually enough.

12. The solution was recycled for 15 minutes by turning the centrifugal pump on.

13. The temperature of water in the storage tank and that of the water in the model tank were taken and compared.

14. The adjusting valve at the discharge meter was set to yield the required flow.

15. The time that the front tip of the mixed layer took to reach each station was recorded.

16. Profiles of the mixed layer were taken at various times.

17. Small crystals of potassium permanganate were thrown into the model tank from the top in order to visualize the general current pattern within the model tank.

18. Pictures were taken of streaks left by the falling crystals of potassium permanganate.

19. The adjusting valve was usually turned off by the time the front tip of the mixed layer was close to the end wall.

20. The model tank was drained overnight.

21. The procedure started at (1) the following morning.

SECTION V

EXPERIMENTAL RESULTS AND ANALYSIS

Basic Data for the Experiments

Twelve experimental runs were performed with the apparatus and procedure described in Section IV. The values of the pertinent physical parameters involved are listed in Table 1 of the Appendix.

General Current Pattern

The general current pattern existing in the model reservoir during a test run is indicated in Figure 7. As water was discharged into the stagnant, density-stratified model tank, an entering layer formed. The entering discharge, q_{in}, caused a hydraulic jump which created a zone of intense turbulence and entrainment. The hydraulic jump acted as a mixing mechanism between the entering fluid and the layers of tank fluid in contact with the hydraulic jump above and below. In this mixing process amounts of fluid from the adjacent fluid were drawn into the hydraulic jump creating reverse currents called q_{zi} as shown in Figure 7. An eddy was consistently formed in the vicinity of the entrance where the mixing between q_{in} and q_{zi} took place. It was possible to observe the mixing zone because the entering fluid had a deep blue color. After the hydraulic jump took place the flow became laminar and stable.

At this point an important distinction should be made between the entering layer and the mixed layer. The entering layer is the flow layer moving in and ahead of the mixed layer. The motion of the entering layer is induced by the entering discharge as it pushes its way into the tank fluid. The thickness of the entering layer increases with horizontal distance from the entering device. The entering layer is formed almost instantaneously as the source is switched on and is the response of the stratified fluid to the sudden increase of pressure created when the source is switched on. The entering layer's thickness grows and its velocity decreases as it moves away from the entrance due to viscous forces.

The mixed layer is a part of the entering layer and consists of fluid mixed by the hydraulic jump. It is therefore a combination of the entering discharge fluid and the ambient fluid. The mixed layer can be distinguished visually from the remainder of the entering layer by dying the discharge fluid; however, the motion of the mixed layer is contiguous to the entering layer and hydrodynamically no distinction can be made.

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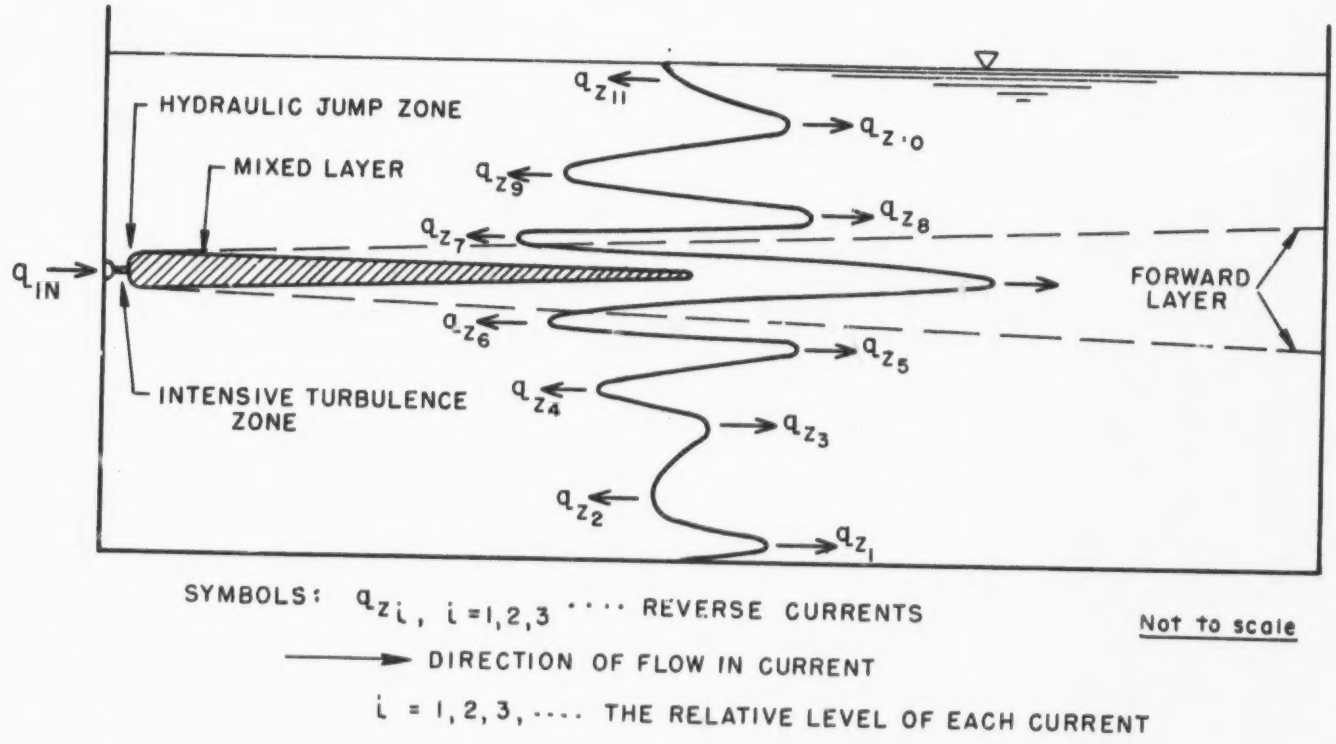


Figure 7 A typical flow distribution observed in the laboratory tank.

From a pollution control point of view the mixed layer is probably more important than the entering layer because the mixed layer is an indicator of the degree of dilution a contaminant would receive by entraining reservoir fluid. This work is concerned with the mixed layer only and deals with the following aspects:

1. The mixed layer length, x_f, for a given time t.
2. The maximum horizontal velocity, U_{max}.
3. Mixed layer thickness, Y, as a function of x and time t.
4. Amount of entrainment, E, taken place up to a given time t.

Figure 8 explains graphically the meaning of the above variables. The following sections report the experimental results concerning the above variables.

Mixed Layer Length

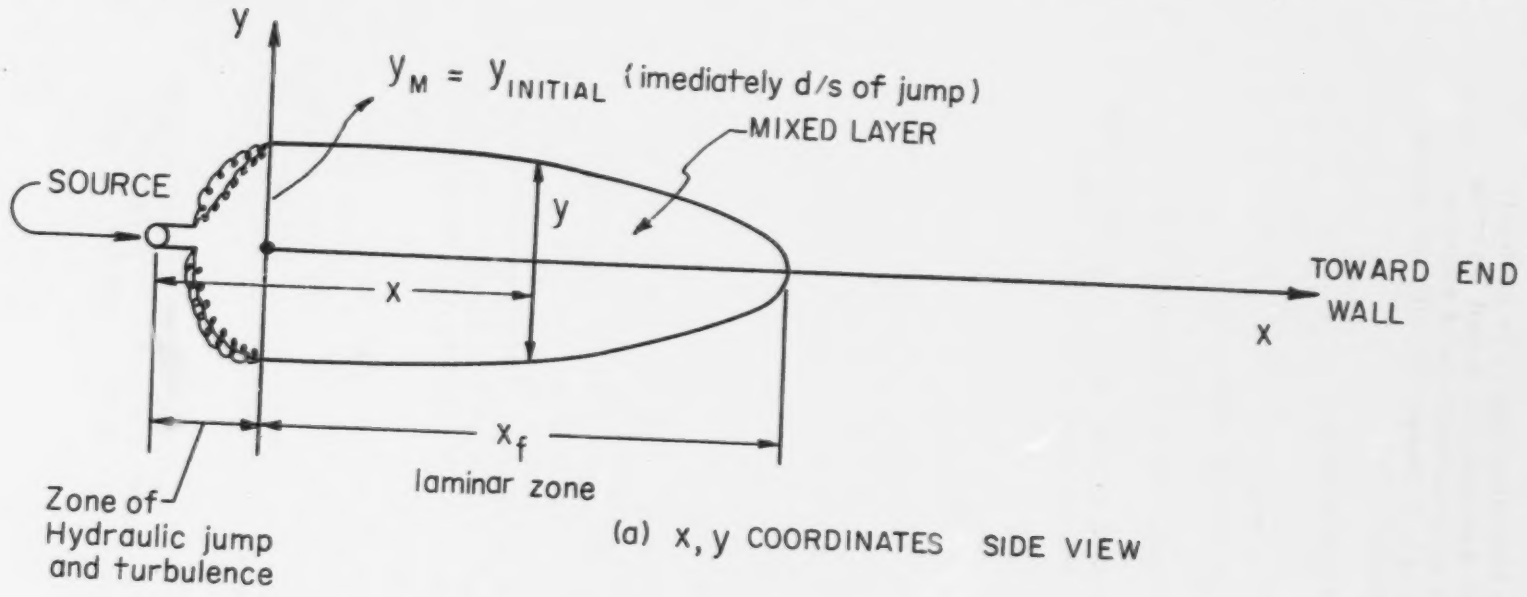
The time taken for the front tip of the mixed layer to reach each one of the 22 established stations was recorded following the procedure described in Section IV. The results for all test runs are listed in Table 2 of the Appendix. The data of Table 2 are plotted in Figure 9 in the form x_f vs. t.

Discussion of Figure 9

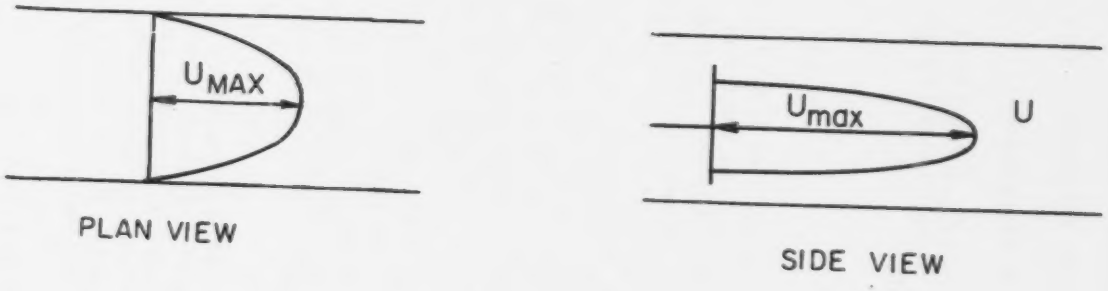
Figure 9 shows on log-log paper the relationship between horizontal distance in cms and time in sec that it took for the front tip of the mixed layer to reach each station. The plotted results show that for each test run there is a well-defined, smooth curve that fits most of the points. The typical curve is approximately linear over the range of points, except for some curvature at the ends. The average slope of the straight line portion of the curves in Figure 9 is . 74, with values ranging from . 72 to . 79. The authors suggest it be taken as . 75, or 3/4.

It is thought that the scatter at the lower end (near the source) is due to experimental errors. At the lower end, an error of ± 1 sec at a point significantly changes the location of that point. The nonlinearity at the upper end is believed to be due to the increased retarding effect of the end wall. The retardation requires a longer time for the front tip to travel a unit distance.

Figure 9 shows that the relationship between the horizontal distance traveled by the front tip of the mixed layer, and time, is parametric with the entering discharge, q_{in}, the linear density gradient, ε, and the water kinematic viscosity, ν.



(a) x, y COORDINATES SIDE VIEW



(b) DEFINITION OF U_{MAX}

FIG. 8 DEFINITION OF THE VARIABLES INVOLVED

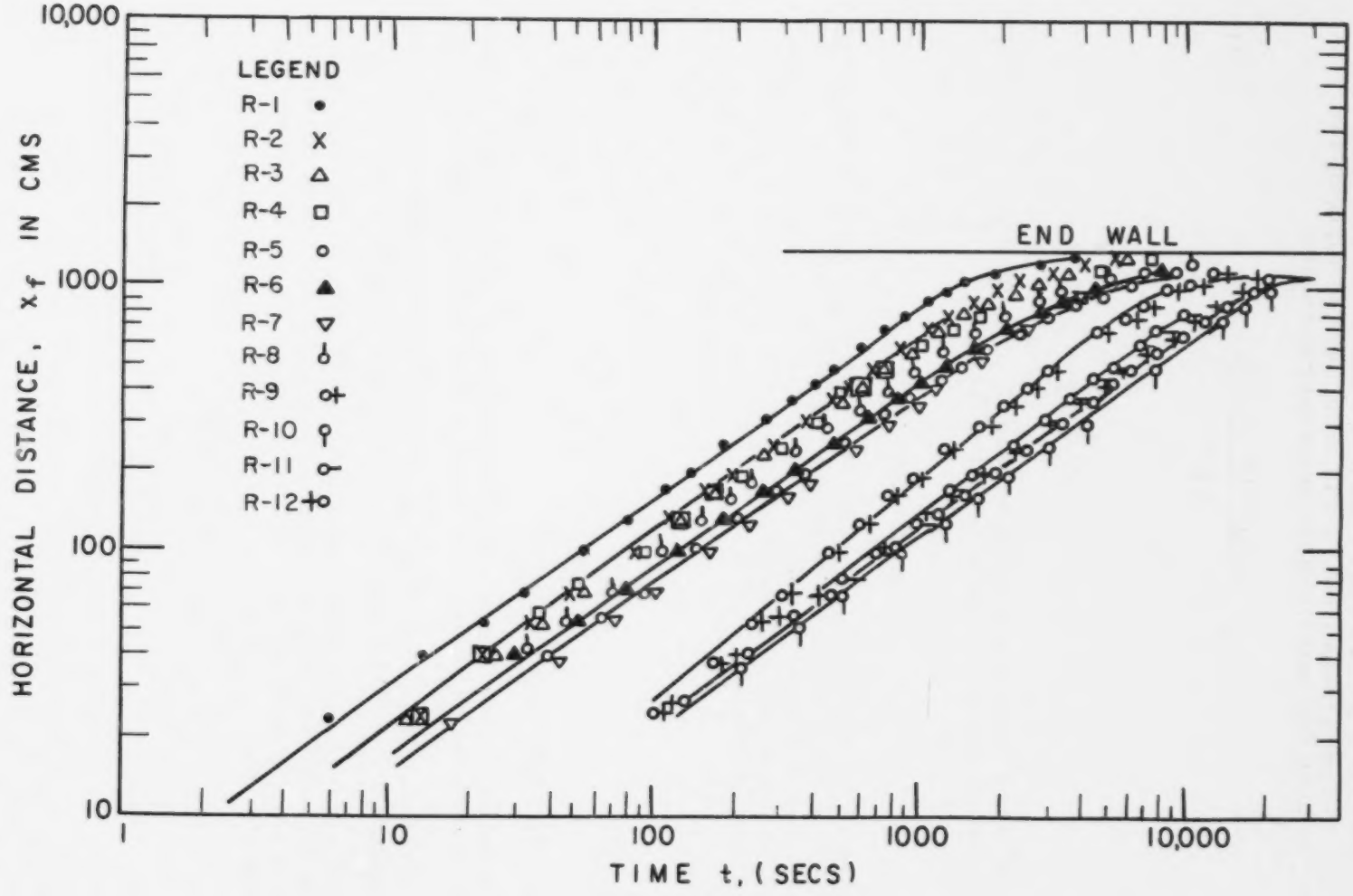


FIG. 9 HORIZONTAL DISTANCE vs TIME

The above three factors were combined to form two dimensionless groups. Of the groups tried, it was found that the following two were the ones that best fitted the experimental results.

$$x^* = \frac{x(\varepsilon g)^{5/12} \nu^{11/18}}{q\, g^{2/9}} \tag{1}$$

and

$$t^* = t(\varepsilon g)^{1/2} \tag{2}$$

The x^* and t^* values are listed in Table 3 in the Appendix. The above values were plotted in Figure 10 on log-log paper.

Discussion of Figure 10

Figure 10 shows on a log-log plot that a well defined relationship exists between x^* and t^*. For all runs, a considerable part of the x^* vs. t^* curves can be synthesized to a single straight line, after which each curve starts to deviate from the straight line portion as it approaches the scaled tank length L^*.

From (1),

$$L^* = \frac{L\,(\varepsilon g)^{5/12}\, \nu^{11/18}}{q\, g^{2/9}} \tag{3}$$

where L = fixed tank length.

Equation (3) shows that the scaled tank length L^* was different for every run, through the dependence of the horizontal scaling factor on q and ε.

As a result, each x^* vs. t^* curve starts to bend from the straight line portion at a different point, until it becomes horizontal at $x^* = L^*$.

The bending is due to the presence of the end wall, which implies that for a tank with an infinite length, the x^* vs. t^* curve must fall on the linear portion of Figure 10. The same figure shows that the end wall effect starts at:

$$x^*_b = .6\, L^* \tag{4}$$

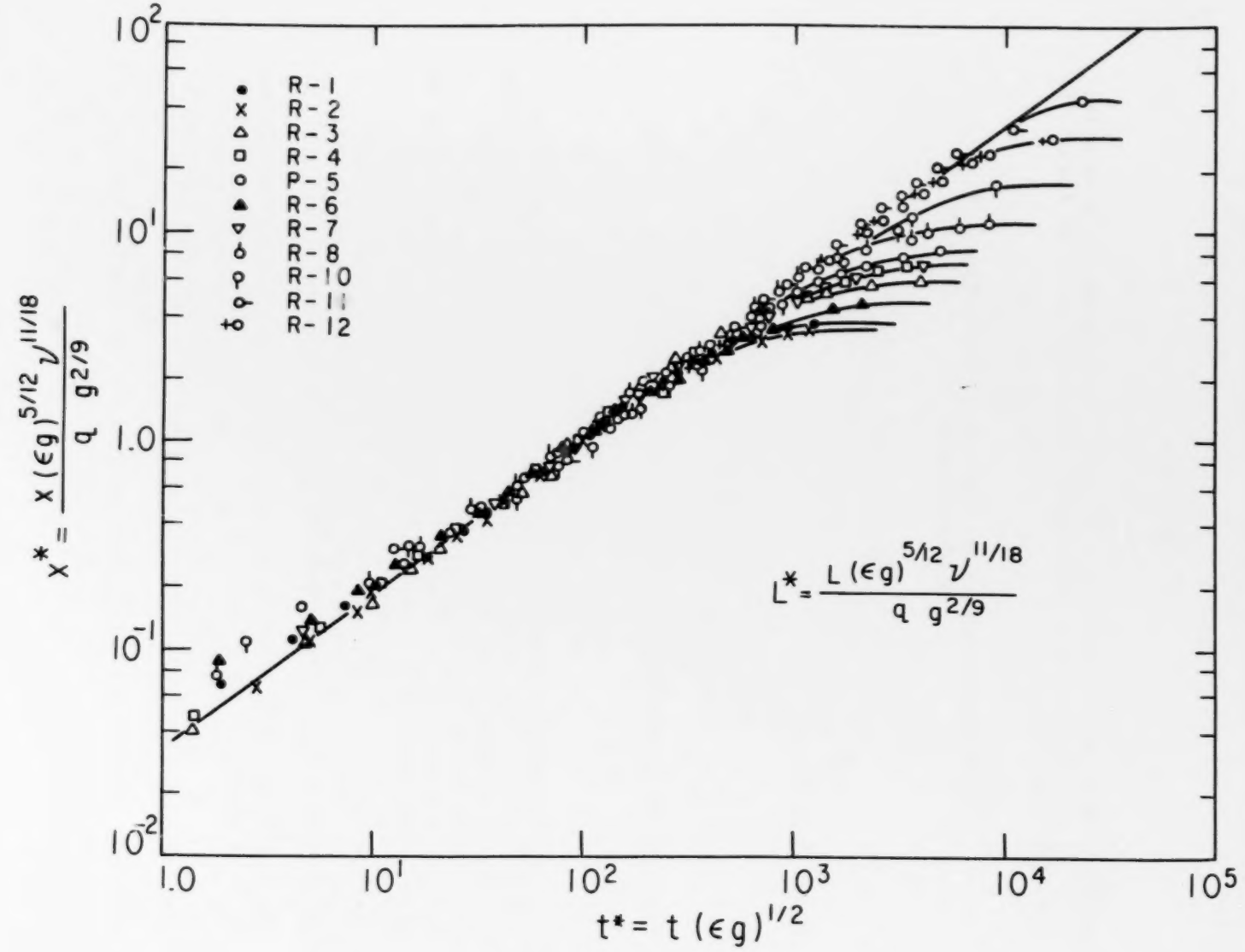


FIG. 10 SCALED HORIZONTAL DISTANCE x^* VS SCALED TIME t^*

The straight line portion of Figure 10 is expressed by Equation (5):

$$x^* = K\,(t^*)^{3/4} \tag{5}$$

where K is a constant equal to 3.16×10^{-2}. Equation (5) is valid for $0 \le x^* \le .6\ L^*$; for larger values of x^*, the actual curve on Figure 10 should be followed.

Maximum Horizontal Velocity of Front Tip of Mixed Layer

$$U_{max} = \frac{dx}{dt} \quad \text{or} \quad U^*_{max} = \frac{dx^*}{dt^*}$$

From Equation (5),

$$U^*_{max} = 3/4\ K\,(t^*)^{-1/4} \tag{6}$$

Equation (6) shows that the velocity decreases with time.

Mixed Layer Thickness

The ideal method of taking the mixed layer profile at a certain time, would have been a photographic one. In the absence of such equipment the profile thickness was measured directly with a measuring tape. In so doing it was impossible to obtain an instantaneous profile. Starting at a given position x_f of the front tip of mixed layer, the thickness of the mixed layer was taken at various stations x such that $0 \le x \le x_f$. The time for each reading was recorded. As the front tip advanced into a new position $x_f + \Delta x$, a new set of readings was taken. From the above two profiles an "instantaneous" profile was obtained by a linear interpolation of both x_f and time.

The resulting mixed layer thickness at each station for a given position of the mixed layer front tip are listed in Table 4 in the Appendix. There is one table for each run. The values listed in Table 4 are plotted in the form Y vs. x_f, where Y and x_f were defined in Figure 8. Figure 11 shows a typical thickness profile.

The main conclusions derived from a thickness profile plot are:

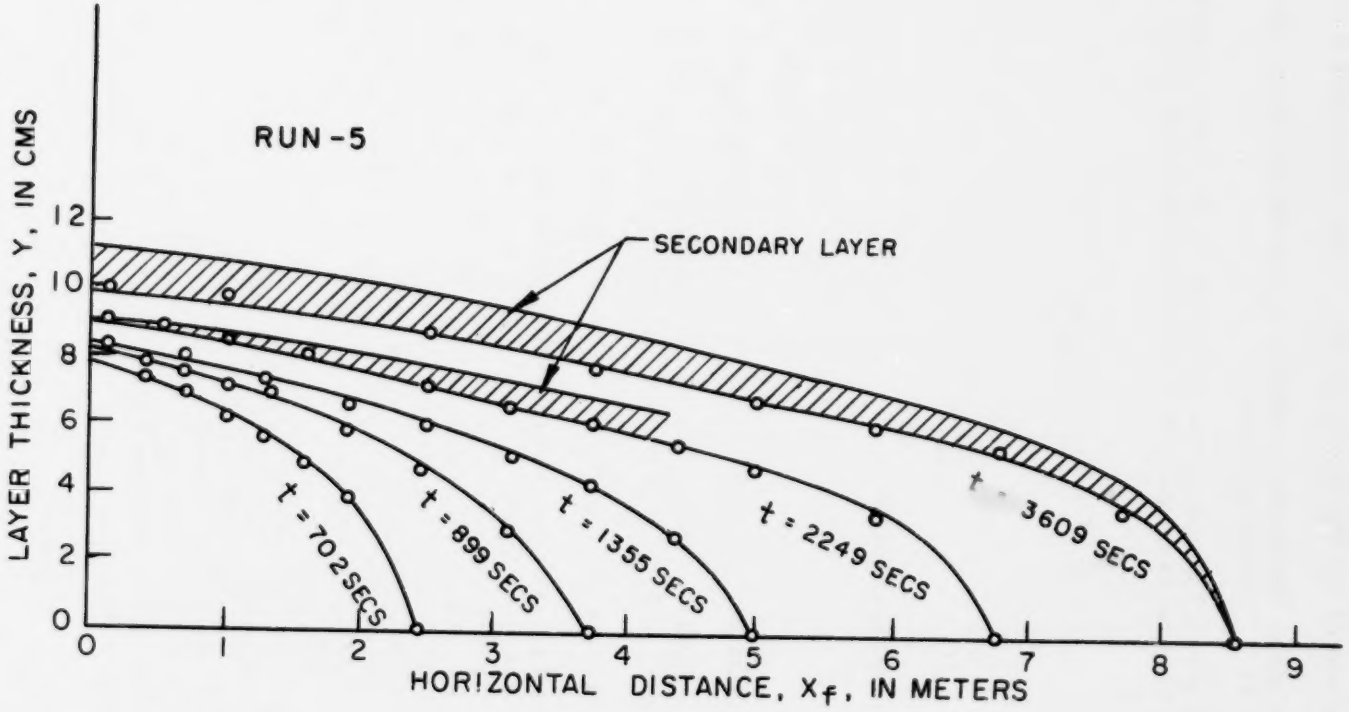


FIG. 11 TYPICAL PROFILE THICKNESS

1. The slope of the thickness profile curve decreases with time.

2. For a given horizontal distance x, the thickness at that point increases as a function of time.

3. For a given Y_m and x_f the thickness Y at a point x such that $0 \leq x \leq x_f$ may be expressed by a parabolic equation of the form

$$Y = Y_m - (Y_m / x_f^{1/2})\, x^{1/2} \tag{7}$$

or

$$Y = Y_m (1 - (x/x_f)^{1/2}) \tag{8}$$

or in a dimensionless form,

$$Y/Y_m = 1 - (x/x_f)^{1/2} \tag{9}$$

An expression for Y_m is given under Entrainment below.

4. The volume in the profile can be expressed in terms of Y_m and x_f as: volume = $2/3\ Y_m\ x_f$, where Y_m and x_f were defined in Figure 8.

5. Conclusions (3) and (4) are valid for values $x^* \leq .6L^*$.

For larger values a practical problem developed in measuring the thickness profile. For $x^* > .6L^*$, a so-called "secondary layer" on top of the main profile, and as shown on Figure 11, started to develop. The secondary layer thickness is not uniform throughout the cross-sectional area of the mixed layer. Instead it is formed by particles of dye being deposited near the side walls. Figure 12 shows schematically the secondary layer. The velocity profile of the secondary layer is the reverse of that in the main layer.

It is thought that the secondary layer is related to currents occurring in the Z-direction. These currents take place due to a difference in temperature between the side walls of the tank and shear in the transverse direction. These local currents are responsible for taking some dye particles from the main mixed layer up against the side walls and therefore carried by the main stream of the reverse currents towards the inlet.

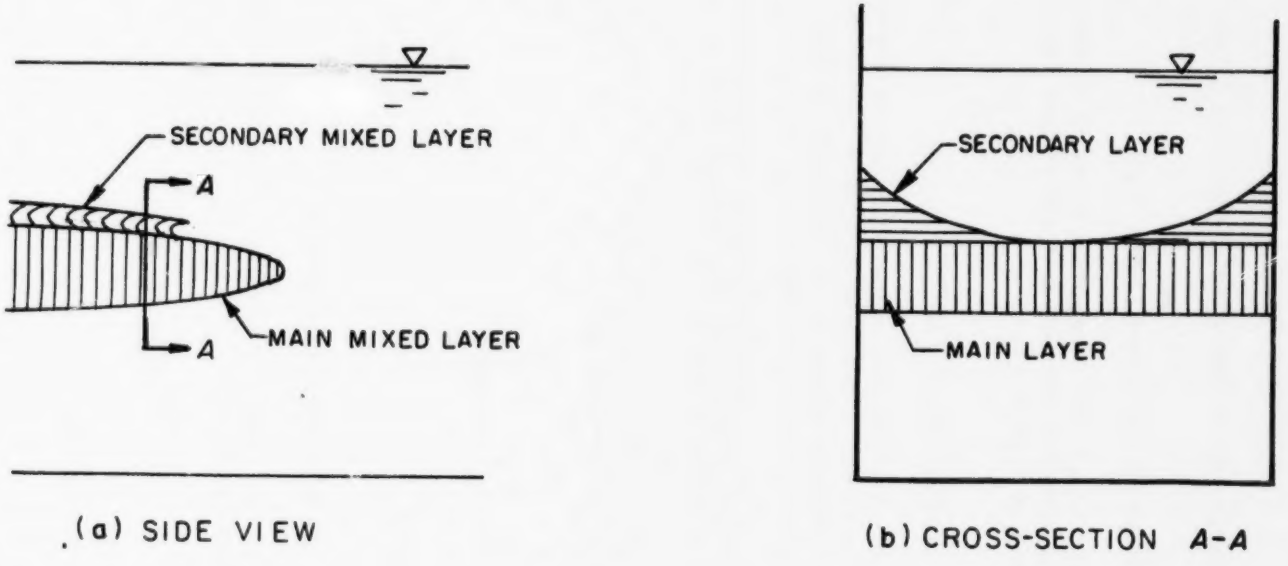


FIG. 12 "SECONDARY LAYER" DESCRIPTION

Entrainment

The amount of entrainment, E, in the mixed layer at a given time t is given by:

$$E = \text{volume in the mixed layer} - q\,t. \tag{10}$$

The volume in the mixed layer was measured from thickness profiles. Table 5 in the Appendix includes the entrainment values for each run. These values are plotted in Figure 13.

Discussion of Figure 13

Figure 13 shows that on a log-log plot a well-defined relationship exists between entrainment and time. This relationship is parametric with the unit discharge q, the density gradient ε, and the viscosity ν. For each run the points fall on a straight line whose slope is 3/4.

The parametric influence of q, ε, and ν upon the entrainment has been reduced by plotting the results of Table 5 in the form E^* vs. t^*.

$$E^* = \frac{E\,(\varepsilon g)^{2/3}\,\nu^{11/18}}{q^{3/2}\,g^{2/9}} \tag{11}$$

and,

$$t^* = t\,(\varepsilon g)^{1/2} \tag{12}$$

Figure 14 shows E^* vs. t^*.

Discussion of Figure 14

The points on Figure 14 fall along two curves, depending upon the outlet conditions Froude number,

$$F = \frac{q}{\sqrt{\varepsilon g}\,h^2}$$

where h = 0.063 cms (slot depth).

For F values between 2.10×10^2 and 9.85×10^2 the points fall on curve A. For F values between $.61 \times 10^2$ and 1.05×10^2 the points fall on curve B. For a given value of t^* curve A gives higher values of E^*.

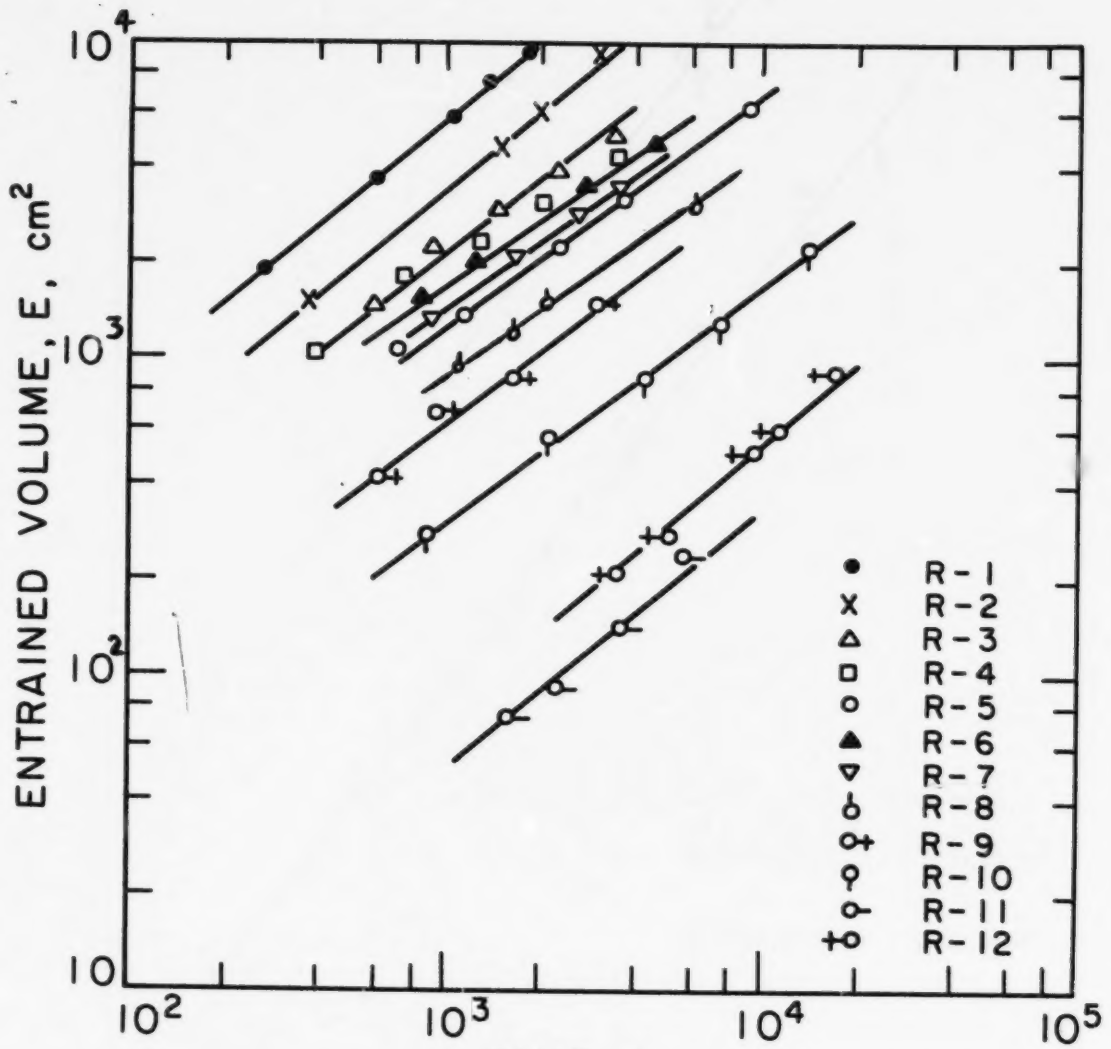


FIG. 13 ENTRAINMENT VS TIME

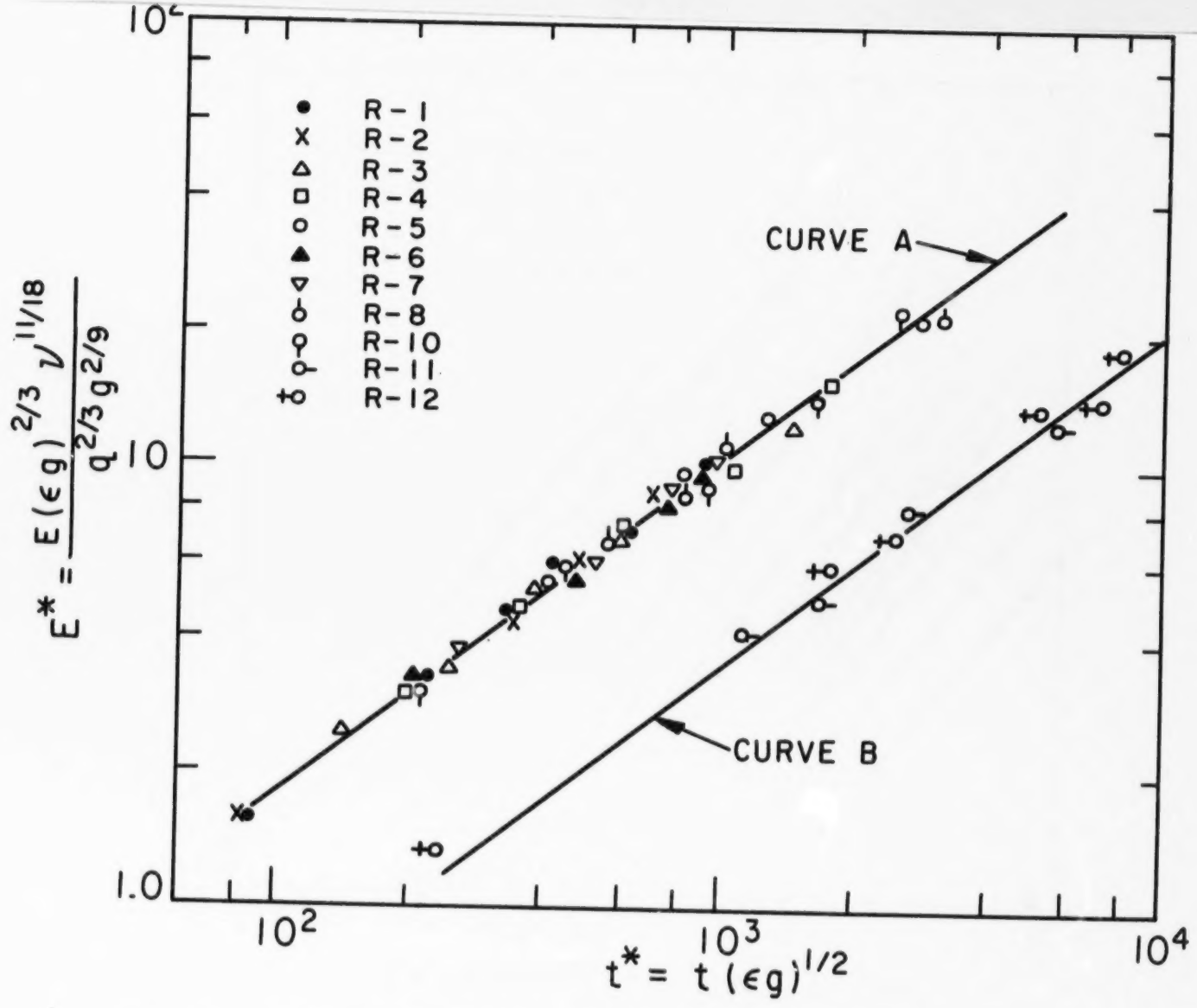


FIG. 14 SCALED ENTRAINMENT E^* VS SCALED TIME t^*

The Froude number is a measure of the intensity of the turbulence produced by the hydraulic jump. The turbulence, in turn, produces entrainment. As a result the entrainment must increase as the Froude number increases.

Curves A and B may be expressed by an equation of the form,

$$E^* = K_1 \, (t^*)^{3/4} \tag{13}$$

where

$K_1 = 5.6 \times 10^2$ for $2.10 \times 10^2 \leq F \leq 9.85 \times 10^2$

or

$K_1 = 2.5 \times 10^2$ for $0.61 \times 10^2 \leq F \leq 1.05 \times 10^2$

Equation (13) shows that for a strong hydraulic jump the entrainment is about twice that of a weak jump for the same t^*.

Equation for Y_m

An equation for Y_m is suggested now from entrainment considerations. The total volume, V, of the mixed layer up to time, t, may be expressed by Equation (14):

$$V = q\,t + E \tag{14}$$

$$V = (2/3)\, Y_m \, x_f \tag{15}$$

Combining Equations (1), (2), and (5)

$$q\,t = \left[\frac{x_f \, (\varepsilon g)^{1/24} \nu^{11/12}}{q^{1/4} \, g^{2/9} \, K} \right]^{4/3} \tag{16}$$

Combining Equations (11), (12), and (13),

$$E = \frac{K_1 \, x_f \, q^{1/2} \, \nu^{11/36}}{K \, (\varepsilon g)^{1/4}} \tag{17}$$

Equation (14) becomes:

$$2/3\ Y_m = x_f^{1/3} \left[\frac{(\varepsilon g)^{1/24} \nu^{11/12}}{q^{1/4} g^{2/9} K} \right]^{4/3} + \frac{K_1 q^{1/2} \nu^{11/36}}{K (\varepsilon g)^{1/4}} \qquad (18)$$

The interesting feature about Equation (18) is that its second term is independent of x_f which means that when $x_f = 0$ then:

$$Y_m = Y_o = \frac{3 K_1 q^{1/2} \nu^{11/36}}{2 K (\varepsilon g)^{1/4}} \qquad (19)$$

The relationship between Equation (19) and the hydraulic jump height should be more carefully studied. At this point it is not possible for the authors to say that Y_o in Equation (19) is the hydraulic jump height. Since no careful record was kept of the jump height, no conclusion can be drawn.

SECTION VI

DISCUSSION OF RESULTS

The average elapsed time required for the front tip of the mixed layer to travel from the injection point to the end wall of the model reservoir was approximately two hours. During that period several factors that were assumed to be constants varied by small amounts. These factors which probably affected the results of Section V are analyzed below.

Temperature Changes

The experiments were conducted in a building whose heating system was incapable of holding the air temperature constant. The changes in the ambient air temperature caused changes in the temperature and kinematic viscosity of the water in the model reservoir. Changes in viscosity introduced changes in the velocity of the front tip. The temperature used herein was the one recorded at the beginning of each experiment. This initial temperature was a crucial one in matching the density of the model tank water at the level of injection to the density of the entering fluid.

Outlet or Injection Level Depth

No outflow from the model tank occurred during each run. The free surface elevation increased at a constant rate given by $dh/dt = q/c$ where c is the surface area of the model tank. The maximum change in free surface elevation was 5 cms which was considered small compared to the initial 50 cms height of the free surface above the injection level. It is believed that any error introduced by the variations in depth is negligible.

Linear Density Gradient

In formulating all the equations in Section V, the value of ε used was the one recorded at the beginning of each run. Figure 15 shows the density profile before and after a typical run. Figure 15 shows that the density becomes uniform around the injection level, and that local variations in ε exist. It is not known yet to what extent these local variations affect the experimental results.

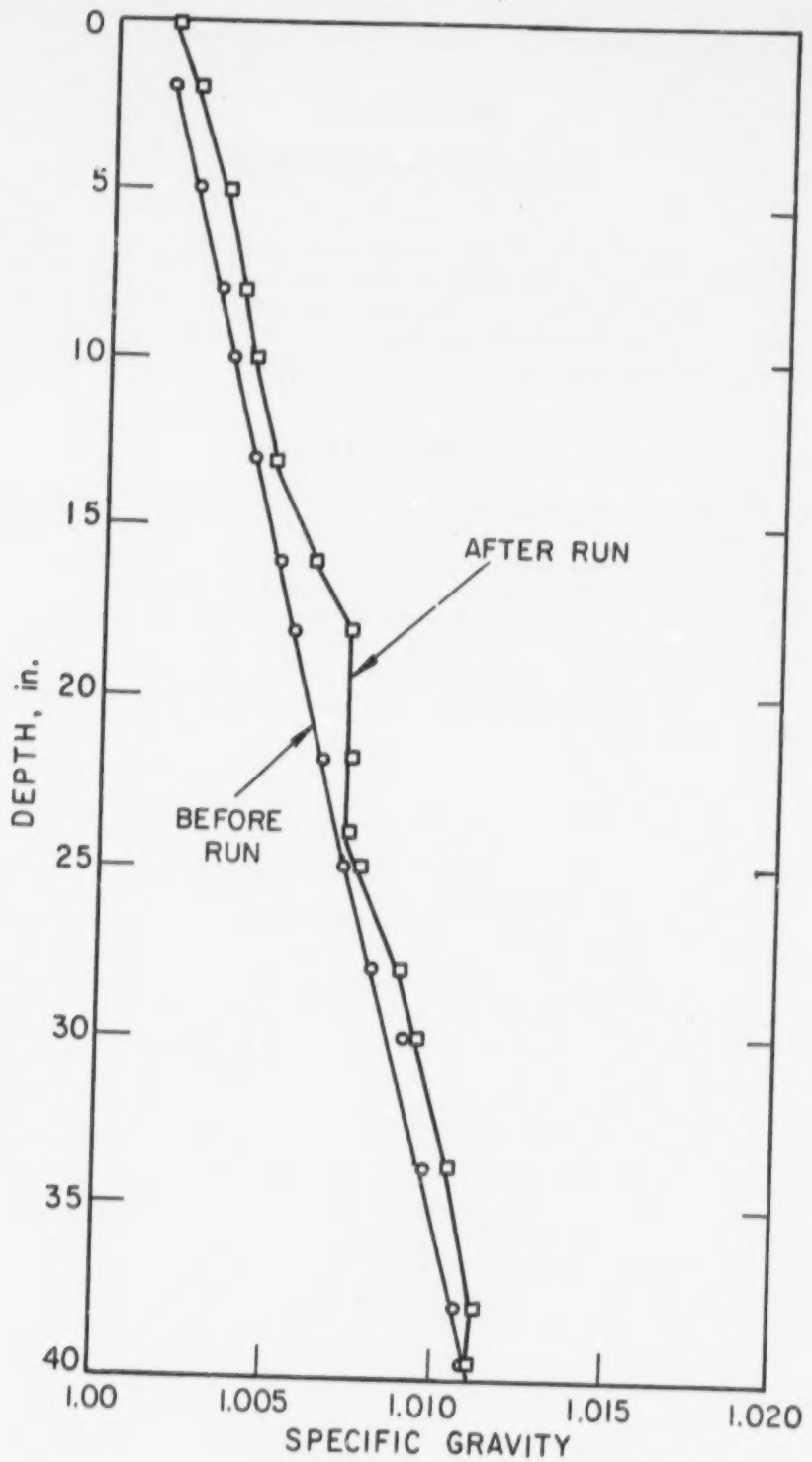


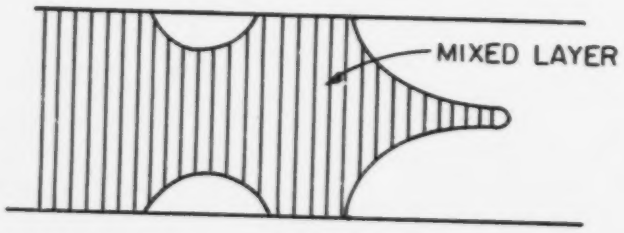
FIG. 15 DENSITY PROFILE SHIFT

Effect of the Side Walls

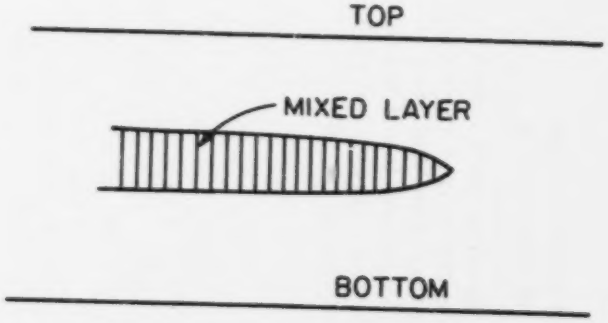
A true two-dimensional flow ir the laboratory tank could not be achieved because of the wall effects. In the area of interest, a parabolic velocity distribution was created across the tank. The shearing force in a horizontal plane produced by the side wall effect was not insignificant in comparison to the shearing force in a vertical plane produced by the reverse currents. The width of the tank was 38 cms while the greatest thickness observed in the mixed layer was approximately 15 cms. A linear approximation of the velocity gives a vertical velocity gradient that is about three times the horizontal velocity gradient.

Blocking Effect of End Wall

As the mixed layer approached the end of the tank, its forward movement was impeded by the end wall. As the blocking effect of the end wall became stronger and stronger, the maximum horizontal velocity of the mixed layer decreased and its thickness increased. As the mixed layer approaches the end wall it seemed to break into a three-dimensional flow. Figure 16 shows some of the patterns observed when the mixed layer was approximately one meter away from the end wall. These patterns were not consistent.

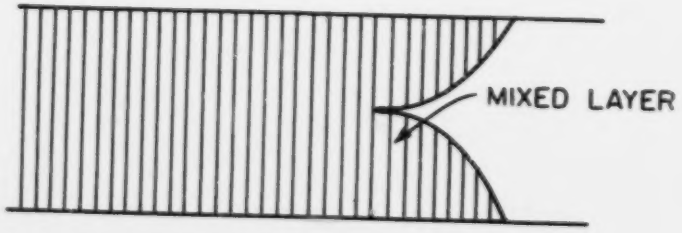


a PLAN VIEW

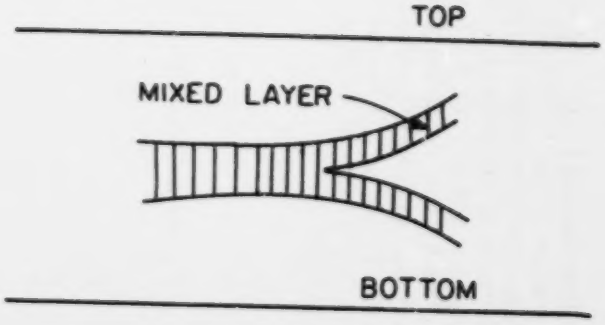


b SIDE VIEW

FIG 4-2o MIXED LAYER (RUN-3) NEAR THE END WALL



a PLAN VIEW



b SIDE VIEW

FIG. 16 MIXED LAYER (RUN-2) NEAR THE END WALL

SECTION VII

ACKNOWLEDGMENTS

This report is based on a thesis submitted in partial fulfillment of the requirements for the degree of Master of Science by Antonio A. Zuluaga-Angel. The report was revised and prepared for publication by R. B. Darden. The research described in this report was under the supervision of Drs. Jorg Imberger and Hugo B. Fischer.

SECTION VIII

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SECTION IX

SYMBOLS

E	=	Entrainment
E^*	=	Scaled entrainment
F	=	Source Froude number based on slot depth, h
g	=	Acceleration due to gravity
h	=	0.063 cms (slot depth)
L	=	Tank length
L^*	=	Scaled tank length
q, q_{in}	=	Discharge measured at inlet
q_{zi}	=	Reverse flows in the model tank, i = 1, 2, . . .
t	=	Time
t^*	=	Scaled time
U_{max}	=	Maximum horizontal velocity of front tip of mixed layer
U^*_{max}	=	Scaled maximum horizontal velocity of front tip of mixed layer
x_f	=	Mixed layer length, measured immediately downstream of hydraulic jump
x^*_f	=	Scaled mixed layer length
x	=	Horizontal distance from inlet
x^*	=	Scaled horizontal distance from inlet
Y	=	Mixed layer thickness

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Y_m	=	Initial thickness of mixed layer
ρ	=	Density of fluid
ρ_o	=	Density at injection level
ε	=	Density gradient $1/\rho_o \, (\Delta\rho/\Delta y)$ (Δy = a difference in elevation)
ν	=	Kinematic viscosity μ/ρ_o
μ	=	Dynamic viscosity

SECTION X

APPENDIX

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TABLE 1

SUMMARY OF BASIC EXPERIMENTAL PARAMETERS FOR EACH RUN

Run No.	Unit Discharge Measured At Inlet q	Density Gradient ε	Temp. T_w	Kinematic Viscosity ν
	cm^2/sec	$10^{-5} cm^{-1}$	^oF	$10^{-5} cm^2/sec$
1	1.93	10.75	62	1100
2	1.38	5.0	68	1014
3	1.38	16.8	66	1043
4	1.38	22.8	64	1072
5	0.83	12.92	68	1014
6	0.89	2.94	69	995
7	0.83	8.24	68	1014
8	0.83	27.5	73	942
9	0.83	10.2	69	995
10	0.29	4.9	68	1014
11	0.277	51.7	69	995
12	0.277	17.75	67	1028

NOTES:

1. Unit discharge = total discharge measured volumetrically/ width of tank.
2. $\varepsilon = \frac{1}{\rho_o} \frac{d\rho}{dy}$

TABLE 2

TIME TAKEN BY THE FRONT TIP OF MIXED LAYER TO REACH EACH STATION

Station Number	Horizontal Distance x_f cms	R-1	R-2	R-3	R-4	R-5	R-6
1	9	-	3.5	3.5	3	3	4
2	24	6	13	12.7	12	13.3	11
3	39	14	23	24.7	23	40	30
3a	55	23	34	38	36	65	50
4	70	32	48	54	51	92	77
5	100	57	84	84	88	147	122
6	131	83	116	119	127	212	187
7	161	110	159	156	164	282	259
8	192	138	194	194	205	357	335
9	253	198	274	276	293	521	490
10	314	263	365	-	389	702	655
11	375	328	453	475	490	899	827
12	436	398	549	585	604	1121	1007
13	496	475	651	697	733	1355	1212
14	586	595	829	905	980	1755	1651
15	680	722	1030	1136	1282	2249	2131
16	770	867	1249	1418	1646	2833	2755
17	863	1037	1527	1778	2259	3609	3312
18	954	1222	1894	2229	2496	4637	4500
19	1045	1473	2354	2792	2996	6115	-
20	1136	1915	3064	3612	3781	9205	-
21	1228	2873	4099	5707	5248	-	-
END WALL	1280	3848	5309	-	7408	-	-

TABLE 2 (Continued)

Station Number	Horizontal Distance x_f cms	R-7	R-8	R-9	R-10	R-11	R-12
1	9	2.5	3.5	14	11	18	-
2	24	17.5	19	112	62	118	-
3	39	-	33	172	210	217	-
3a	55	-	49	241	354	272	348
4	70	-	70	311	523	-	-
5	100	164	108	465	865	709	811
6	131	226	149	615	1240	986	1147
7	161	308	186	775	1670	1279	-
8	192	384	231	935	2119	1579	1839
9	253	541	332	1278	3142	2221	-
10	314	711	450	1654	4276	2914	3384
11	375	901	592	2069	-	3645	-
12	436	1081	760	2522	-	4479	5147
13	496	-	941	2965	7703	5209	6120
14	586	1693	1222	-	-	6529	7694
15	680	2143	1610	4802	-	8359	9500
16	770	2732	2056	5895	13916	-	11557
17	863	3550	2666	6928	16600	-	-
18	954	-	3455	8240	-	-	16832
19	1045	-	5375	98711	-	-	19802
20	1136	-	7055	-	-	-	-
21	1228	-	10600	-	-	-	-
END WALL	1280	-	-	-	-	-	-

TABLE 3

x^* AND t^* VALUES

Station Number	R-1		R-2		R-3	
	x^*	t^*	x^*	t^*	x^*	t^*
1	.025	-	.024	.776	.041	1.42
2	.068	1.94	.065	2.88	.109	5.15
3	.11	4.2	.105	5.1	.177	10.0
3a	.16	7.45	.150	7.55	.250	15.4
4	.20	10.4	.19	10.65	.318	21.8
5	.28	18.5	.27	18.6	.455	34
6	.37	26.9	.35	25.8	.596	48
7	.46	35.6	.43	35.3	.732	63
8	.54	44.7	.52	43.0	.874	78.5
9	.72	64.0	.68	61	1.15	112
10	.89	85.5	.85	81	1.43	-
11	1.06	106	1.01	100	1.71	192
12	1.24	129	1.18	122	1.98	237
13	1.41	154	1.34	144	2.26	283
14	1.66	193	1.58	184	2.67	367
15	1.93	234	1.84	228	3.09	461
16	2.19	281	2.08	277	3.50	575
17	2.45	336	2.33	340	3.93	720
18	2.70	395	2.58	420	4.34	900
19	2.97	476	2.82	524	4.75	1132
20	3.22	620	3.07	680	5.17	1465
21	3.49	930	3.31	910	5.59	2320
END WALL	3.64	1250	3.46	1175	5.82	-

TABLE 3 (Continued)

Station Number	R-4		R-5		R-6	
	x^*	t^*	x^*	t^*	x^*	t^*
1	.048	1.41	.060	1.06	.032	.68
2	.127	5.65	.16	4.7	.086	1.87
3	.207	10.8	.26	14.1	.14	5.1
3a	.291	17	.37	23	.198	8.5
4	.371	24	.47	32.5	.25	13.1
5	.53	41.5	.67	52	.36	20.8
6	.69	60	.87	75	.47	31.8
7	.85	77.4	1.07	100	.58	44
8	1.02	96.6	1.28	126	.69	57
9	1.34	138	1.68	185	.91	83.4
10	1.66	183	2.09	248	1.13	111
11	1.99	230	2.50	318	1.35	141
12	2.31	285	2.90	390	1.57	171
13	2.63	346	3.30	480	1.78	206
14	3.10	461	3.90	620	2.11	282
15	3.60	605	4.53	795	2.45	363
16	4.08	776	5.13	1000	2.77	470
17	4.57	1065	5175	1275	3.11	564
18	5.06	1180	6.35	1640	3.43	765
19	5.54	1410	6.96	2160	376	-
20	6.02	1780	7.57	3260	4.09	-
21	6.51	2470	8.18	-	4.42	-
END WALL	6.78	3500	8.52	-	4.61	-

TABLE 3 (Continued)

Station Number	R-7		R-8		R-10	
	x^*	t^*	x^*	t^*	x^*	t^*
1	.049	.71	.078	1.85	.12	2.4
2	.132	5	.21	9.9	.32	13.6
3	.214	-	.34	17.0	.52	46
3a	.302	-	.48	30	.73	77.5
4	.38	-	.61	50	.93	114
5	.55	46.5	.87	70	1.33	190
6	.72	64.4	1.14	130	1.74	272
7	.88	87.5	1.40	170	2.14	366
8	1.056	109	1.67	170	2.55	465
9	1.39	154	2.20	260	3.36	690
10	1.72	202	2.73	360	4.18	940
11	2.06	256	3.26	500	4.98	-
12	2.40	307	3.79	640	5.80	-
13	2.73	-	4.31	680	6.60	1690
14	3.22	480	5.10	840	7.80	-
15	3.74	610	5.90	1000	9.00	-
16	4.23	775	6.70	1300	10.24	3040
17	4.75	1000	7.51	1550	11.48	3630
18	5.25	-	8.3	2100	12.69	-
19	5.75	-	9.1	3500	13.90	-
20	6.25	-	9.9	4200	15.11	-
21	6.75	-	10.7	6000	16.33	-
END WALL	7.04	-	11.19	8300	17.02	-

TABLE 3 (Continued)

Station Number	R-11		R-12	
	x^*	t^*	x^*	t^*
1	.32	12.8	.21	-
2	.85	84	.55	-
3	1.38	1545	.90	-
3a	1.95	193	1.26	145
4	2.48	-	1.61	-
5	3.54	505	2.3	339
6	4.64	700	3.0	478
7	5.70	910	3.7	-
8	6.80	1123	4.4	767
9	8.96	1580	5.8	-
10	11.11	2075	7.2	1410
11	13.27	2600	8.6	-
12	15.4	3190	10.0	2150
13	17.6	3710	11.4	2560
14	20.74	4650	13.5	3210
15	24.1	5950	15.6	3970
16	27.2	-	17.7	4825
17	30.6	-	19.8	-
18	33.8	-	21.9	7030
19	37	-	24	8260
20	40.2	-	26	-
21	43.5	-	28.2	-
END WALL	45.3	-	29.4	-

TABLE 4

SERIES PROFILE VALUES

R-1

Profile Time sec	265	600	1050	1350
Horizontal Distance cms		Thickness cms		
9	12.05	13.95	-	-
24	12.05	13.95	-	-
39	12.05	13.95	14.25	16.25
55	12.05	13.95	14.25	16.25
70	12.70	13.65	14.60	16.50
100	12.25	12.40	13.70	14.30
131	11.15	12.10	12.75	14.30
161	9.20	12.10	13.05	15.00
192	10.50	11.45	12.70	14.30
253	9.20	10.65	12.25	13.50
314	7.00	9.85	11.45	12.75
375	4.10	8.90	11.10	12.10
436	0.0	8.30	10.65	11.80
496		7.05	10.00	11.45
586		4.45	8.75	10.50
680		0.0	7.30	9.55
770			5.75	8.25
863			2.85	7.30
954			0.0	5.25
1045				0.0
1136				
1228				
1280				

TABLE 4 (Continued)

R-2

Profile Time sec	1500	2000	3200
Horizontal Distance cms		Thickness cms	
9	-	-	-
24	-	-	-
39	-	-	-
55	-	-	-
70	11. 60	11. 80	-
100	11. 20	11. 7	14. 8
131	10. 7	11. 6	14. 4
161	10. 6	11. 6	14. 6
192	10. 2	11. 2	14. 0
253	9. 85	11. 0	14. 2
314	9. 36	10. 9	13. 5
375	9. 00	10. 4	13. 3
436	8. 25	9. 90	13. 2
496	7. 97	9. 5	12. 4
586	6. 75	8. 6	12. 0
680	5. 5	7. 5	22. 5
770	2. 8	6. 5	10. 6
863	0. 0	5. 4	10. 4
954		0. 0	9. 5
1045			8. 2
1136			0. 0
1228			
1280			

TABLE 4 (Continued)

R-3

Profile Time sec	590	900	1450	2300	3450
Horizontal Distance cms			Thickness cms		
9	-	-	-	-	-
24	-	-	-	-	-
39	-	-	-	-	-
55	-	-	-	-	-
70	-	-	-	-	-
100	8. 20	8. 70	9. 60	10. 80	12. 30
131	7. 70	8. 20	9. 15	10. 70	11. 73
161	7. 00	7. 94	8. 80	10. 20	11. 65
192	6. 35	7. 48	8. 30	9. 60	11. 65
253	5. 26	7. 00	8. 10	9. 10	11. 53
314	4. 33	6. 36	7. 72	8. 73	11. 15
375	1. 31	5. 1	7. 00	8. 50	10. 62
436	0. 0	4. 67	6. 50	8. 03	10. 64
586		3. 70	5. 82	7. 05	10. 20
680		0. 0	4. 40	5. 93	9. 55
770			3. 20	4. 63	8. 74
863			0. 0	3. 13	8. 00
954				0. 0	7. 10
1045					5. 45
1136					4. 35
1228					0. 0
1280					

TABLE 4 (Continued)

R-4

Profile Time sec	710	1250	2000	3500
Horizontal Distance cms			Thickness cms	
9	-	-	-	-
24	-	-	-	-
39	-	-	-	-
55	-	8.10	-	-
70	-	8.30	9.30	-
100	8.10	8.20	9.21	10.80
131	8.00	8.00	9.17	10.70
161	7.10	8.20	9.00	10.70
192	6.62	7.87	8.87	10.67
253	5.93	7.52	8.71	10.45
314	4.88	7.00	8.25	10.40
375	3.75	6.54	8.20	10.12
436	1.56	6.39	7.97	10.0
496	0.0	4.98	7.14	9.73
586		2.40	6.70	9.28
680		0.0	5.66	8.67
770			2.54	7.70
863			0.0	7.55
954				6.15
1045				4.03
1136				0.0
1228				
1280				

TABLE 4 (Continued)

R-5

Profile Time sec	700	1150	2220	3600	9000
Horizontal Distance cms	Thickness cms				
9	7.8	8.0	8.5	9.5	14.1
24	7.7	7.9	8.4	9.5	14.1
39	7.6	7.8	8.4	9.4	14.0
55	7.5	7.7	8.3	9.4	14.0
70	7.4	7.6	8.3	9.3	13.9
100	7.0	7.4	8.2	9.3	13.8
131	6.4	7.0	8.2	9.2	13.7
161	5.8	6.8	8.0	9.1	13.4
192	5.2	6.3	7.9	9.0	13.3
253	3.4	5.5	7.4	8.8	13.1
314	0.0	4.6	6.8	8.4	12.8
375		3.0	6.4	8.0	12.6
436		0.0	5.8	7.4	12.5
496			5.0	7.0	12.0
586			3.8	6.3	11.6
680			0.0	5.8	11.0
770				4.5	10.2
863				0.0	9.6
954					8.4
1045					6.5
1136					0.0
1228					
1280					

TABLE 4 (Continued)

R-6

Profile Time sec	830	1200	2700	4600
Horizontal Distance cms	Thickness cms			
9	-	-	-	-
24	-	-	-	-
39	-	-	-	-
55	-	-	-	-
70	9.00	9.2	10.7	-
100	8.25	8.6	10.6	11.0
131	7.5	8.4	10.20	11.00
161	6.70	7.9	10.3	11.0
192	6.23	7.7	10.0	11.0
253	4.90	6.50	9.4	10.60
314	1.60	5.80	8.6	10.30
375	0.0	4.20	8.10	10.10
436		3.20	7.50	10.20
496		0.0	7.20	9.25
586			5.70	8.75
680			3.90	8.25
770			0.0	7.30
863				4.60
954				0.0
1045				
1136				
1228				
1280				

TABLE 4 (Continued)

R-7

Profile Time sec	900	1650	2600	3500
Horizontal Distance cms	Thickness cms			
9	12.40	12.60	13.6	15.0
24	10.85	12.50	13.5	14.8
39	11.40	12.3	13.4	14.7
55	11.1	12.0	13.3	14.6
70	10.8	11.8	13.2	14.4
100	9.6	11.2	13.0	14.3
131	8.6	10.8	12.9	13.8
161	8.0	10.4	12.5	13.6
192	7.4	9.6	12.4	13.0
253	6.4	8.6	11.0	12.5
314	5.0	7.7	10.5	11.3
375	0.0	6.7	9.0	10.5
436		6.1	8.4	10.0
496		5.0	7.35	9.0
586		0.0	7.0	8.0
680			4.75	7.0
770				6.0
863				0.0
954				
1045				
1136				
1228				
1280				

TABLE 4 (Continued)

R-10

Profile Time sec	880	2150	4300	7600	1400
Horizontal Distance cms			Thickness cms		
9	6.40	8.0	7.4	8.6	8.3
24	7.05	7.8	7.3	8.3	8.2
39	6.5	7.8	7.1	8.1	8.4
55	5.4	7.7	7.0	7.8	8.3
70	4.6	7.5	6.9	7.7	8.2
100	0.0	6.1	6.8	7.6	8.1
131		5.3	6.7	7.5	8.0
161		4.0	6.5	7.4	7.8
192		0.0	6.2	7.3	7.7
253			6.0	6.9	7.6
314			0.0	6.5	7.5
375				6.3	7.4
436				6.1	7.1
496				0.0	6.5
586					5.5
680					5.0
770					0.0
863					
954					
1045					
1136					
1228					
1280					

TABLE 4 (Continued)

R-11

Profile Time sec	1550	2300	3600	5600
Horizontal Distance cms	Thickness cms			
9	3.8	4.15	4.5	4.8
24	3.7	4.1	4.5	4.7
39	3.5	4.0	4.4	4.6
55	3.4	3.7	4.3	4.5
70	3.2	3.7	4.1	4.4
100	2.9	3.3	4.0	4.3
131	2.2	2.7	3.9	4.2
161	1.60	2.4	3.5	4.1
192	0.0	2.0	2.7	3.8
253		0.0	2.5	3.5
314			2.2	3.0
375			0.0	2.5
436				2.2
496				0.0
586				
680				
770				
863				
954				
1045				
1136				
1228				
1280				

TABLE 5

ENTRAINMENT VALUES

	E cm^2	E^*	t sec	t^*
R-1				
$F = 9.45 \times 10^2$	1440	1.55	265	88
	3100	3.3	600	220
	5600	6.0	1050	330
	6500	7.0	1350	420
	9300	10.0	1810	640
R-2				
$F = 9.85 \times 10^2$	1500	1.55	365	83
	4600	4.4	1500	340
	6000	6.0	2000	480
	9000	8.6	3200	700
R-3				
$F = 5.40 \times 10^2$	1400	3.4	590	245
	2200	5.3	900	380
	2850	6.8	1450	590
	3900	9.5	2300	900
	5000	12.5	3450	1450
R-4				
$F = 4.65 \times 10^2$	1050	3.0	390	195
	1800	4.8	710	350
	2300	7.4	1250	600
	3000	10.0	2000	1050
	4300	15.5	3500	1750

TABLE 5 (Continued)

	E cm^2	E^*	t sec	t^*
R-5				
$F=3.72 \times 10^2$	1060	4.6	700	330
	1350	5.6	1150	400
	2220	9.6	2220	810
	3200	13	3600	1280
	5400	23	9000	2800
R-6				
$F=7.75 \times 10^2$	1500	2.4	830	150
	2000	2.9	820	205
	3500	5.1	2700	480
	4600	6.7	4600	770
R-7				
$F=4.65 \times 10^2$	1350	4.3	900	260
	2100	6.7	1650	520
	2800	9.0	2600	770
	3500	11.2	3500	960
R-8				
$F=2.52 \times 10^2$	930	6.7	1060	550
	1200	8.6	1600	830
	1530	11	1920	1000
	3070	22	6000	3100

TABLE 5 (Continued)

	E cm^2	E^*	t sec	t^*
R-10 $F=2.10 \times 10^2$	270	3.0	880	210
	560	6.10	2150	450
	850	9.20	4300	830
	1200	12.0	7600	1000
	2200	23.0	14000	3100
R-11 $F=.61 \times 10^2$	73	4.1	1550	1100
	90	5.0	2300	1650
	140	7.9	3600	2650
	240	12.0	5600	5800
R-12 $F=1.05 \times 10^2$	210	5.9	3500	1800
	270	7.2	5000	2500
	500	13.0	9300	5200
	600	15.0	11500	7000
	750	19.0	17000	8000